



National Numeracy Learning Progression

Numeracy Progression
Appendices

Contents

Appendix 1. The evidence base for development	3
Appendix 2. Key references – Number sense and algebra	4
Appendix 3. Key references – Measurement and geometry.....	12
Appendix 4. Key references – Statistics and probability.....	15
Appendix 5. Numeracy experts consulted during development.....	17

Appendix 1. The evidence base for development

While a wide range of literature was considered in the development of the National Numeracy Learning Progression, three references were of particular significance. These are as follows:

National Research Council 2001, *Adding it up: Helping children learn mathematics*, J Kilpatrick, J Swafford & B Findell (eds), Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, National Academy Press, Washington, DC.

The evidence provided in *Adding it up* underpins the sequencing of the content of the Australian Curriculum: Mathematics. This National Research Council (USA) report describes what is known about how children learn to be numerate. It provides a synthesis of research with strong academic credentials, based on research that meets standards of relevance, soundness and generalisability (p. 23). Despite its publication date of 2001, it is still considered current and continues to be frequently cited.

In describing what is known about how children learn to be numerate, the report identifies that much more research has been conducted in the domain of number than in the development of spatial sense or data sense. Indeed, within the report, only one chapter is dedicated to research that meets the standards of the synthesis and that pertains to students' learning about space or helping students learn about data and chance. Extra comment on these areas was sought from those engaged in research in these emerging domains. Where research meeting the same standards of relevance, soundness and generalisability was not available, the Australian Curriculum was used to determine the initial sequencing of levels within the progression.

National Numeracy Review (Australia) & Stanley, GV & Council of Australian Governments Human Capital Working Group 2008, *National Numeracy Review report*, COAG, Canberra.

The National Numeracy Review report has also been influential to the learning progressions. It confirms that research associated with number sense holds a dominant position in the existing literature. It recognises that the balance of research was weighted towards primary education, especially the early years. It also notes, 'One of the striking features of the numeracy literature is the lack of creative research about the necessary, or possible, mathematical content most likely to support rich numeracy practices' (p. 10). This finding acknowledges the paucity of evidence about applications of numeracy across learning areas other than Mathematics.

XH Sun, B Kaur & J Novotná (eds) 2015, *Conference proceedings of ICMI Study 23: Primary mathematics study on whole numbers*, June 2015, Macau, China.
www.umac.mo/fed/ICMI23/proceedings.html

To ensure the most up-to-date research was employed in the creation of the numeracy progression, research drawn from the twenty-third International Commission on Mathematical Instruction (ICMI) 2015 study, titled 'Primary mathematics study on whole numbers', was used to supplement the comprehensive research synthesis in *Adding it up*.

Listings of the literature considered for the elements of the National Numeracy Learning Progression are provided in appendices 2–4.

It is acknowledged that new evidence and research will emerge, perhaps through this project, that may challenge the sequencing of content in the Australian Curriculum. New evidence will be reported in ACARA's annual process for monitoring the effectiveness of the Australian Curriculum, for consideration in future Australian Curriculum evaluations and reviews.

Appendix 2. Key references – Number sense and algebra

Quantifying

Anghileri, J 2008, *Developing Number Sense: Progression in the middle years*, Continuum International Publishing Group, London.

Anghileri, J 2006, *Teaching Number Sense*, 2nd edn, Bloomsbury Academic, London.

Aprea, C 2016, *International Handbook of Financial Literacy*, 1st 2016 edn, Springer, Singapore.

Australian Securities and Investments Commission 2011, *National Consumer and Financial Literacy Framework*, ASIC.

Ellemor-Collins, D & Wright, R 2007, 'Assessing pupil knowledge of the sequential structure of numbers', *Educational & Child Psychology*, 24(2), pp. 54–63.

Fuson, KC 1988, *Children's Counting and Concepts of Number*, Springer-Verlag, New York.
Fuson, KC 1992, 'Research on whole number addition and subtraction', in DA Grouws (ed.),

Handbook of Research on Mathematics Teaching and Learning, Macmillan, New York, Toronto, pp. 243–275.

Gelman, R & Gallistel, C 1978, *The Child's Understanding of Number*, Harvard University Press, Cambridge, MA.

Gould, P 2017, 'Mapping the acquisition of the number word sequence in the first year of school', *Mathematics Education Research Journal*, 29(1), pp. 93–112.

Hunting, RP 2003, 'Part-whole number knowledge in preschool children', *The Journal of Mathematical Behavior*, 22(3), pp. 217–235.

McIntosh, A, Reys, B, Reys, R, Bana, J & Farrell, B 1997, *Number sense in school mathematics: Student performance in four countries*, MASTEC, Perth.

Mix, KS, Huttenlocher, J & Levine, SC 2002, *Quantitative Development in Infancy and Early Childhood*, Oxford University Press, Oxford, UK.

Mix, KS, Sandhofer, CM & Baroody, AJ 2005, 'Number words and number concepts: the interplay of verbal and nonverbal quantification in early childhood', in RV Kail (ed.), *Advances in Child Development and Behavior*, vol. 33, Academic Press, New York.

Mulligan, J, Bobis, J & Francis, C 1999, 'Insights into early numeracy: the count me in too project', *Australian Primary Mathematics Classroom*, 4(1), pp. 22–26.

Organisation for Economic Co-operation and Development, Programme for International Student Assessment & OECD2004, PISA 2003 Assessment Framework: mathematics, reading, science and problem solving knowledge and skills, OECD, Paris.

Pepper, KL & Hunting, RP 1998, 'Preschoolers' counting and sharing', *Journal for Research in Mathematics Education*, pp. 164–183.

Reid, K and Andrews, N 2016, 'Fostering understanding of early numeracy development', Australian Council for Educational Research, retrieved from <http://research.acer.edu.au/monitoring_learning/29>, accessed on 28 September 2016.

Rogers, AN 2014, 'Investigating whole number place value assessment in Years 3–6: creating an evidence-based developmental progression', doctoral dissertation, RMIT University, Victoria.

Stacey, K & Steinle, V 1999, 'Understanding decimals: the path to expertise', *Making the difference: proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia Incorporated, MERGA*, Sydney, pp. 446–453.

Sousa, DA 2008, *How the Brain Learns Mathematics*, Corwin Press, Thousand Oaks, CA.

Steffe, LP 1992, 'Learning stages in the construction of the number sequence', in J Bideaud, C Meljac & JP Fischer (eds), *Pathways to Number*, Lawrence Erlbaum, Hillsdale, NJ, pp. 82–98.

Wright, RJ 1989, 'Numerical development in the kindergarten year: a teaching experiment', doctoral dissertation, University of Georgia, GA.

Wynn, K 1990, 'Children's understanding of counting', *Cognition*, 36(2), pp. 155–193.

Additive strategies, Multiplicative strategies and Understanding money

Anghileri, J 1989, 'An investigation of young children's understanding of multiplication', *Educational Studies in Mathematics*, 20, pp. 367–385.

Anghileri, J 2001, *Principles and Practices in Arithmetic Teaching*, Open University Press, Buckingham.

Aprea, C 2016, *International Handbook of Financial Literacy*, 1st 2016 edn, Springer, Singapore.

Australian Securities and Investments Commission 2011, *National Consumer and Financial Literacy Framework*, ASIC.

Baroody, AJ, Eiland, M & Thompson, B 2009, 'Fostering at-risk preschoolers' number sense', *Early Education and Development*, 20(1), pp. 80–128.

Baroody, AJ, Feil, Y & Johnson, AR 2007, 'An alternative reconceptualization of procedural and conceptual knowledge', *Journal for Research in Mathematics Education*, pp. 115–131.

- Battista, M 2012, *Cognition-Based Assessment and Teaching of Place Value: Building on students' reasoning*, Heinemann, Portsmouth, NH.
- Butterworth, B 2005, 'The development of arithmetical abilities', *Journal of Child Psychology and Psychiatry*, 46(1), pp. 3–18.
- Chandler, CC & Kamii, C 2009, 'Giving change when payment is made with a dime: the difficulty of tens and ones', *Journal for Research in Mathematics Education*, 40(2), pp. 97–118.
- Clark, FB & Kamii, C 1996, 'Identification of multiplicative thinking in children in grades 1–5', *Journal for Research in Mathematics Education*, 27(1), pp. 41–51.
- Denvir, B & Brown, M 1986a, 'Understanding of number concepts in low attaining 7–9 year olds: Part I. Development of descriptive framework and diagnostic instrument', *Educational Studies in Mathematics*, 17, pp. 15–36.
- Denvir, B & Brown, M 1986b, 'Understanding of number concepts in low attaining 7–9 year olds: Part II. The teaching studies', *Educational Studies in Mathematics*, 17, pp. 143–164.
- Dowker, A 2008, 'Individual differences in numerical abilities in preschoolers', *Developmental Science*, 11(5), pp. 650–654.
- Fuson, KC 1992, 'Research on whole number addition and subtraction', in DA Grouws (ed.), *Handbook of Research on Mathematics Teaching & Learning*, MacMillan, New York, pp. 243–275.
- Geary, DC 2011, 'Cognitive predictors of achievement growth in mathematics: a five year longitudinal study', *Developmental Psychology*, 47(6), pp. 1539–1552.
- Howell, SC & Kemp, CR 2010, 'Assessing preschool number sense: skills demonstrated by children prior to school entry', *Educational Psychology*, 30(4), pp. 411–429.
- Jordan, NC, Kaplan, D, Ramineni, C & Locuniak, MN 2009, 'Early math matters: kindergarten number competence and later mathematics outcomes', *Developmental Psychology*, 45(3), p. 850.
- Gelman, R & Gallistel, CR 1978, *The Child's Understanding of Number*, Harvard University Press, Cambridge, MA.
- Gould, P, Outhred, L & Mitchelmore, MC 2006a, 'One-third is three-quarters of one-half', in P Grootenboer, R Zevenbergen & M Chinnappan (eds), *Identities, cultures and learning spaces: proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia*, vol. 1, MERGA, Adelaide, pp. 262–269.
- Hunting, RP 2003, 'Part-whole number knowledge in preschool children', *The Journal of Mathematical Behavior*, 22(3), pp. 217–235.

- Klein, T & Beishuizen, M 1994, 'Assessment of flexibility in mental arithmetic', in JEH LuitVan (ed.), *Research on Learning and Instruction in Kindergarten and Primary School*, Graviatt Publishing Company, Doetinchem, the Netherlands, pp. 125–152.
- McIntosh, A, Reys, B, Reys, R, Bana, J & Farrell, B 1997, *Number Sense in School Mathematics: Student performance in four countries*, MASTEC, Perth.
- Mulligan, J 1992, 'Children's solutions to multiplication and division word problems: a longitudinal study', *Mathematics Education Research Journal*, 4(1), pp. 24–41.
- Mulligan, J & Wright, R 2000, 'Interview-based assessment of early multiplication and division', *Proceedings of the 24th International Conference for the Psychology of Mathematics Education*, 4, pp. 17–24, Hiroshima, Japan.
- Mulligan, JT & Mitchelmore, MC 1997, 'Young children's intuitive models of multiplication and division', *Journal for Research in Mathematics Education*, 28(3), pp. 309–330.
- OECD 2013, *PISA 2012 Assessment and Analytical Framework: mathematics, reading, science, problem solving and financial literacy*, OECD Publishing, Paris.
- Pepper, KL & Hunting, RP 1998, 'Preschoolers' counting and sharing', *Journal for Research in Mathematics Education*, pp. 164–183.
- Rogers, AN 2014, 'Investigating whole number place value assessment in Years 3–6: creating an evidence-based developmental progression', doctoral dissertation, RMIT University, Victoria.
- Ross, SH 1989, 'Parts, wholes and place value: a developmental view', *Arithmetic Teacher*, 36(6), pp. 47–51.
- Rousham, L 2003, 'The empty number line: a model in search of a learning trajectory?' in I Thompson (ed.), *Enhancing Primary Mathematics Teaching*, Open University Press, Philadelphia, pp. 29–39.
- Sousa, DA 2008, *How the Brain Learns Mathematics*, Corwin Press, Thousand Oaks, CA.
- Stacey, K & Steinle, V 1999, 'Understanding decimals: the path to expertise', in *Making the difference: proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia Incorporated*, pp. 446–453.
- Steffe, LP 1992, 'Learning stages in the construction of the number sequence', in J Bideaud, C Meljac & J Fischer (eds), *Pathways to Number: children's developing numerical abilities*, Lawrence Erlbaum, Hillsdale, NJ, pp. 83–88.
- Steffe, LP 1992, 'Schemes of action and operation involving composite units', *Learning and Individual Differences*, 4, pp. 259–309.
- Steffe, LP 1994, 'Children's multiplying schemes', in G Harel & J Confrey (eds), *The Development of Multiplicative Reasoning in the Learning of Mathematics*, State University of New York Press, Albany, NY, pp. 3–39.

Steffe, LP & Cobb, P 1998, 'Multiplicative and divisional schemes', *Focus on Learning Problems in Mathematics*, 20(1), pp. 45–61.

Steffe, LP, Cobb, P & von Glasersfeld, E 1988, *Construction of Arithmetical Meanings and Strategies*, Springer-Verlag, New York.

Steffe, LP, von Glasersfeld, E, Richards, J & Cobb, P 1983, *Children's Counting Types: philosophy, theory, and application*, Praeger, New York.

Thompson, I 1995, 'The role of counting in the idiosyncratic mental calculation algorithms of young children', *European Early Childhood Education Research Journal*, 3(1), pp. 5–16.

van der Ven, SH, Straatemeier, M, Jansen, BR, Klinkenberg, S & van der Maas, HL 2015, 'Learning multiplication: an integrated analysis of the multiplication ability of primary school children and the difficulty of single digit and multidigit multiplication problems', *Learning and Individual Differences*, 43, pp. 48–62.

Wright, RJ 1991, 'The role of counting in children's numerical development', *The Australian Journal of Early Childhood*, 16(2), pp. 43–48.

Wright, RJ 1994, 'A study of the numerical development of 5-year-olds and 6-year-olds', *Educational Studies in Mathematics*, 26, pp. 25–44.

Wynn, K 1990, 'Children's understanding of counting', *Cognition*, 36(2), pp. 155–193.

Young-Loveridge, J 2005, 'Fostering multiplicative thinking using array-based materials', *The Australian Mathematics Teacher*, 61(3), p. 34.

Number patterns and algebraic thinking

Aprea, C 2016, *International Handbook of Financial Literacy*, 1st 2016 edn, Springer, Singapore.

Australian Securities and Investments Commission 2011, *National Consumer and Financial Literacy Framework*, ASIC.

Berg, W van der and D van Eerde 1985, *Kwantiwijzer (Number diagnostics)*, SVO/Erasmus University, Rotterdam.

Blanton, M, Stephens, A, Knuth, E, Gardiner, AM, Isler, I & Kim, JS 2015, 'The development of children's algebraic thinking: the impact of a comprehensive early algebra intervention in third grade', *Journal for Research in Mathematics Education*, 46(1), pp. 39–87.

Booker, G & Windsor, W 2010, 'Developing algebraic thinking: using problem-solving to build from number and geometry in the primary school to the ideas that underpin algebra in high school and beyond', *Procedia: Social and Behavioral Sciences*, 8, pp. 411–419.

Chimoni, M & Pitta-Pantazi, D 2015, 'Connections between algebraic thinking and reasoning processes', in *CERME 9: Ninth Congress of the European Society for Research in Mathematics Education*, pp. 398–404, Prague, Czech Republic.

8

- Chua, BL & Hoyles, C 2014, 'Generalisations of linear figural patterns in secondary school mathematics', *The Mathematics Educator*, 15(2), pp. 1–30.
- Clements, DH 1999, 'Subitizing: What is it? why teach it?' *Teaching Children Mathematics*, 5(7), pp. 400–405.
- de Vries, E, Cole, A & Warren, E 2009, 'Closing the gap: myths and truths behind subitisation', *Australasian Journal of Early Childhood*, 34(4), p. 46.
- Fitzhugh, JI 1978, 'The role of subitising and counting in the development of the young children's conception of small numbers', *Dissertation Abstracts International*, 40.
- Fosnot, CT & Dolk, M 2001, *Young Mathematicians at Work: Constructing multiplication and division*, Heinemann, Portsmouth, NH.
- Fuson, KC 1992, 'Research on whole number addition and subtraction', in DA Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, New York, Toronto, pp. 243–275.
- Hatano, G 1982, 'Learning to Add and Subtract: a Japanese perspective', in JP Carpenter, JM Moser and TA Romberg (eds), *Addition and Subtraction: a cognitive perspective*, Lawrence Erlbaum, Hillsdale, NJ, pp. 211–224.
- International Network on Financial Education 2012, *Financial Education in Schools*, INFE.
- Kaput, JJ 2008, 'What is algebra? What is algebraic reasoning?' in JJ Kaput, DW Carraher & ML Blanton (eds), *Algebra in the Early Grades*, Lawrence Erlbaum Associates, New York, NY, pp. 5–17.
- Kaufman, EL, Lord, MW, Reese, TW & Volkman, J 1949, 'The discrimination of visual number', *American Journal of Psychology*, 62, pp. 498–525.
- McIntosh, A, Reys, B, Reys, R, Bana, J & Farrell, B 1997, *Number Sense in School Mathematics: Student performance in four countries*, MASTEC, Perth.
- Mulligan, J & Mitchelmore, M 2009, 'Awareness of pattern and structure in early mathematical development', *Mathematics Education Research Journal*, 21(2), pp. 33–49.
- OECD 2013, *PISA 2012 Assessment and Analytical Framework: mathematics, reading, science, problem solving and financial literacy*, OECD Publishing, Paris.
- Olive, J 2000, 'Children's number sequences: an explanation of Steffe's constructs and an extrapolation to rational numbers of arithmetic', *The Mathematics Educator*, 11(1).
- Papic, MM, Mulligan, JT & Mitchelmore, MC 2011, 'Assessing the development of preschoolers' mathematical patterning', *Journal for Research in Mathematics Education*, 42(3), pp. 237–268.
- Steffe, LP 1992, 'Learning stages in the construction of the number sequence', in J Bideaud, C Meljac & JP Fischer (eds), *Pathways to Number*, Lawrence Erlbaum, Hillsdale, NJ, pp. 82–98.

Stephens, A & Gardiner, AM 2015, 'The development of children's algebraic thinking: the impact of a comprehensive early algebra', *Journal for Research in Mathematics Education*, 46(1), pp. 39–87.

Wirtz, R 1980, *New Beginnings: a guide to think, talk, and read math centers for beginners*, Curriculum Development Associates, Monterey, CA.

Wolter, G, van Kempen, H & Wijhuizen, GJ 1987, 'Quantification of small numbers of dots: subitizing or pattern recognition?' *American Journal of Psychology*, 100, pp. 225–237.

Fractions, Decimals and Percentages

Aprea, C 2016, *International Handbook of Financial Literacy*, 1st 2016 edn, Springer, Singapore.

Armstrong, BE & Larson, CN 1995, 'Students' use of part-whole and direct comparison strategies for comparing partitioned rectangles', *Journal of Research in Mathematics Education*, 26(1), pp. 2–19.

Australian Securities and Investments Commission 2011, *National Consumer and Financial Literacy Framework*, ASIC.

Ball, DL 1993, 'Halves, pieces, and twos: Constructing and using representational contexts in teaching fractions', in TP Carpenter, E Fennema & TA Romberg (eds), *Rational Numbers: An integration of research*, Erlbaum, Mahwah, NJ, pp.157–195.

Baratta, W, Price, B, Stacey, K, Steinle, V & Gvozdenko, E 2010, 'Percentages: The effect of problem structure, number complexity and calculation format, Shaping the future of mathematics education: proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia Incorporated, MERGA, Fremantle, pp. 61–68.

Behr, M, Harel, G, Post, T & Lesh, R 1993, 'Rational numbers: toward a semantic analysis – emphasis on the operator construct', in TP Carpenter, E Fennema & TA Romberg (eds), *Rational Numbers: An integration of research*, Erlbaum, Mahwah, NJ, pp. 13–47.

Bruce, C, Chang, D, Flynn, T & Yearley, S 2013, 'Foundations to learning and teaching fractions: Addition and subtraction', retrieved 4 July 2014.

Carpenter, TP, Corbitt, MK, Kepner, HS, Lindquist, MM & Reys, RE 1981, 'Results from the Second Mathematics Assessment of the National Assessment of Educational Progress', National Council of Teachers of Mathematics, Washington, DC.

Charalambous, C & Pitta-Pantazi, D 2005, 'Revisiting a theoretical model on fractions: implications for teaching and research', paper presented at the 29th Conference of the International Group for the Psychology of Mathematics Education, Melbourne, Australia, retrieved 4 September 2011 from <www.emis.de/proceedings/PME29/PME29CompleteProc/PME29Vol2_Adl_Fre.pdf>

Clarke, D & Roche, A 2011, 'Some advice for making the teaching of fractions a research-based, practical, effective and enjoyable experience in the middle years', *Australian Catholic*

10

University, retrieved 8 September 2011 from
<<http://gippslandtandlcoaches.wikispaces.com/file/view/fractions.pdf>>

Clarke, DM & Roche, A 2009, 'Students' fraction comparison strategies as a window into robust understanding and possible pointers for instruction', *Educational Studies in Mathematics*, 72, pp. 127–138.

Confrey, J 2012, 'Articulating a learning sciences foundation for learning trajectories in the CCSS-M', in L Van Zoest, JJ Lo & J Kratky (eds), *Proceedings of the 34th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Kalamazoo, MI, pp. 2–22.

Confrey, J, Maloney, AP, Wilson, PH & Nguyen, KH 2010, 'Understanding over time: the cognitive underpinnings of learning trajectories', paper presented at the annual meeting of the American Education Research Association, Denver, CO.

DeWolf, M, Bassok, M & Holyoak, KJ 2015, 'Conceptual structure and the procedural affordances of rational numbers: relational reasoning with fractions and decimals', *Journal of Experimental Psychology: General*, vol. 144, no. 1, pp. 127–150.

Gelman, R & Gallistel, C 1978, *The Child's Understanding of Number*, Harvard University Press, Cambridge, MA.

Gould, P 2008, 'Children's quantitative sense of fractions', doctoral dissertation, Macquarie University, Sydney.

Gould, P, Outhred, L & Mitchelmore, MC 2006a, 'One-third is three-quarters of one-half', in P Grootenboer, R Zevenbergen & M Chinnappan (eds), *Identities, cultures and learning spaces: proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia*, vol. 1, MERGA, Adelaide, pp. 262–269.

Hackenberg, A 2007, 'Units coordination and the construction of improper fractions: a revision of the splitting hypothesis', *Journal of Mathematical Behavior*, 26(1), pp. 27–47.

Hunting, RP 2003, 'Part-whole number knowledge in preschool children', *Journal of Mathematical Behavior*, 22(3), pp. 217–235.

International Network on Financial Education 2012, *Financial Education in Schools*, INFE.

Kamii, C & Kysh, J 2006, 'The difficulty of "length x width": is a square the unit of measurement?' *Journal of Mathematical Behavior*, 25, pp. 105–115.

Kieren, T 1980, 'The rational number construct: its elements and mechanisms', in TE Kieren (ed.), *Recent Research on Number Learning*, pp. 125–149.

Kieren, TE 1993, 'Rational and fractional numbers: from quotient fields to recursive understanding', in TP Carpenter, E Fennema & TA Romberg (eds), *Rational numbers: An integration of research*, Lawrence Erlbaum Associates, Inc., Hillsdale, NJ, pp. 49–84.

Lamon, S 1999, *Teaching Fractions and Ratios for Understanding*, Lawrence Erlbaum Associates, Mahwah, NJ.

Lamon, S 2001, 'Presenting and representing: From fractions to rational numbers', in A Cuoco & F Curcio (eds), *The roles of representations in school mathematics – 2001 yearbook*, NCTM, Reston, pp. 146–165.

McIntosh, A, Reys, B, Reys, R, Bana, J & Farrell, B 1997, *Number Sense in School Mathematics: Student performance in four countries*, MASTEC, Perth.

National Research Council 2001, *Adding It Up: Helping children learn mathematics*, J Kilpatrick, J Swafford & B Findell (eds), Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, National Academy Press, Washington, DC.

Nunes, T & Bryant, P 2009, 'Paper 3: Understanding rational numbers and intensive quantities', Nuffield Foundation, London.

OECD 2013, *PISA 2012 Assessment and Analytical Framework: mathematics, reading, science, problem solving and financial literacy*, OECD Publishing, Paris.

Pepper, KL & Hunting, RP 1998, 'Preschoolers' counting and sharing', *Journal for Research in Mathematics Education*, pp. 164–183.

Stacey, K, & Steinle, V 1999, 'Understanding decimals: the path to expertise', in *Making the difference: proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia Incorporated*, pp. 446–453.

Steffe, L & Olive, J 2010, *Children's Fractional Knowledge*, Springer Science and Business Media, New York, NY.

White, P, & Mitchelmore, M 2005, 'Teaching percentage as a multiplicative relationship' in *Building connections: Research, theory and practice, proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia Incorporated*, pp. 805–814.

Appendix 3. Key references – Measurement and geometry

Measurement

Battista, MT, Clements, DH, Arnoff, J, Battista, K & Borrow, CVA 1998, 'Students' spatial structuring of 2D arrays of squares', *Journal for Research in Mathematics Education*, 29(5), pp. 503–532.

Bragg, P & Outhred, NL 2004, 'A measure of rulers: the importance of units in a measure', in MJ Høines & AB Fuglestad (eds), *Proceedings of the 28th annual conference of the International Group for the Psychology of Mathematics Education*, vol. 2, Program Committee, Bergen, Norway, pp. 159–166.

Cheeseman, J, McDonough, A & Ferguson, S 2014, 'Investigating young children's learning of mass measurement', *Mathematics Education Research Journal*, 26(2), p. 131.

Clements, DH & Sarama, J 2014. *Learning and teaching early math: The learning trajectories approach*. Routledge, New York.

Curry, M, Mitchelmore, M & Outhred, L 2006, 'Development of children's understanding of length, area, and volume principles', in J Novotná, H Moraová, M Krátká & N Stehlíková (eds), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*, vol. 2, PME, Prague, pp. 377–384.

Inhelder, B, Sinclair, H & Bovet, M 1974, *Learning and the Development of Cognition*, Harvard University Press, Cambridge, MA.

Outhred, L & McPhail, D 2000, 'A framework for teaching early measurement', in J Bana & A Chapman (eds), *Proceedings of the 23rd Annual Conference of the Mathematics Education Research Group of Australasia*, vol. 2, MERGA, Sydney, Australia, pp. 487–494.

Outhred, L & Mitchelmore, MC 2000, 'Young children's intuitive understanding of rectangular area measurement', *Journal for Research in Mathematics Education*, 31, pp. 144–167.

Geometric properties

Battista, MT, Clements, DH, Arnoff, J, Battista, K & Borrow, CVA 1998, 'Students' spatial structuring of 2D arrays of squares', *Journal for Research in Mathematics Education*, pp. 503–532.

Burger, WF & Shaughnessy, JM 1986, 'Characterizing the van Hiele levels of development in geometry', *Journal for Research in Mathematics Education*, pp. 31–48.

Fox, TB 2000, 'Implications of research on children's understanding of geometry', *Teaching Children Mathematics*, 6(9), p. 572.

Freksa, C 1992, 'Using orientation information for qualitative spatial reasoning', Springer, Berlin Heidelberg, pp. 162–178.

Khan, S, Francis, K & Davis, B 2015, 'Accumulation of experience in a vast number of cases: enactivism as a fit framework for the study of spatial reasoning in mathematics education', *ZDM*, 47(2), pp. 269–279.

Lehrer, R, Fennema, E, Carpenter T & Ansell, E 1994, 'Review of NCRSME research', *NCRSME Research Review: the teaching and learning of mathematics*, National Center for Research in Mathematical Sciences Education Research, Madison, Wis., pp. 10–13.

Mitchelmore, MC & White, P 2000, 'Development of angle concepts by progressive abstraction and generalisation', *Educational Studies in Mathematics*, 41(3), pp. 209–238.

Mulligan, J, Mitchelmore, M & Prescott, A 2005, 'Case studies of children's development of structure in early mathematics: a two-year longitudinal study', in PME Conference, July, vol. 29, no. 4, p. 4.

Mulligan, J, Prescott, A & Mitchelmore, MC 2004, 'Children's development of structure in early mathematics', in MJ Høines & AB Fuglestad (eds), Proceedings of the 28th annual conference of the International Group for the Psychology of Mathematics Education, vol. 3, pp. 393–400, PME, Bergen, Norway.

Owens, KD & Clements, MK 1998, 'Representations in spatial problem solving in the classroom', *The Journal of Mathematical Behavior*, 17(2), pp. 197–218.

Piaget, J, Inhelder, B & Szeminski, A 1960, *The Child's Conception of Geometry*, Routledge and Kegan Paul, London.

Position and location

Bruce, CD, Moss, J, Sinclair, N, Whiteley, W, Okamoto, Y, McGarvey, L & Davis, B 2013, 'Early years spatial reasoning: learning, teaching, and research implications', NCTM research precession: linking research and practice.

Ehrlich, SB, Levine, SC & Goldin-Meadow, S 2006, 'The importance of gesture in children's spatial reasoning', *Developmental Psychology*, 42(6), pp. 1259–1268.

Norman, DK 1980, 'A comparison of children's spatial reasoning: rural Appalachia, suburban, and urban New England', *Child Development*, 51(1), pp. 288–291.

Petty, MR & Rule, AC 2008, 'Effective materials for increasing young children's spatial and mapping skills', *Journal of Geoscience Education*, 56(1), p. 5.

Time

Australian Education Council Curriculum 1991, *A National Statement on Mathematics for Australian Schools*, Curriculum Corporation, Melbourne.

Booker, G, Bond, D, Briggs, J & Davey, G 1997, *Teaching Primary Mathematics*, 2nd edn, Longman, Australia.

Bruce, CD, Moss, J, Sinclair, N, Whiteley, W, Okamoto, Y, McGarvey, L & Davis, B 2013, 'Early years spatial reasoning: learning, teaching, and research implications', NCTM research precession: linking research and practice.

Clements, D & Bright, G 2003, *Learning and Teaching Measurement*, National Council of Teachers of Mathematics, USA.

Ehrlich, SB, Levine, SC & Goldin-Meadow, S 2006, 'The importance of gesture in children's spatial reasoning', *Developmental Psychology*, 42(6), pp. 1259–1268.

Friedman, WJ 1982, 'Conventional time concepts and children's structuring of time', in WJ Friedman (ed.), *The Developmental Psychology of Time*, Academic Press, New York, pp. 171–208.

Norman, DK 1980, 'A comparison of children's spatial reasoning: rural Appalachia, suburban, and urban New England', *Child Development*, 51(1), pp. 288–291.

Piaget, J 1969, *The Child's Conception of Time*, Routledge and K Paul, London.

Queensland Studies Authority 2005, *About Measurement, The State of Queensland*, retrieved from <www.qcaa.qld.edu.au/downloads/p_10/kla_maths_info_measurement.pdf>, accessed 27 July 2016.

Appendix 4. Key references – Statistics and probability

Chance

Bryant, P & Nunes, T 2012, *Children's Understanding of Probability: a literature review* (summary report), Nuffield Foundation.

Burrill, G & Elliott, PC 2006, *Thinking and Reasoning with Data and Chance*, National Council of Teachers of Mathematics, Reston, Va.

Callingham, R & Watson, JM 2004, 'Measuring statistical literacy', *Journal of Applied Measurement*, 6(1), pp. 19–47.

Garfield, JB & Ben-Zvi, D 2008, *Developing Students' Statistical Reasoning: connecting research and teaching practice*, Springer, Dordrecht; London.

Konold, C & Kazak, S 2008, 'Reconnecting data and chance', *Technology Innovations in Statistics Education*, 2(1).

MacGillivray, H & Pereira-Mendoza, L 2011, 'Teaching statistical thinking through investigative projects', in *Teaching Statistics in School Mathematics: challenges for teaching and teacher education*, Springer Netherlands, pp. 109–120.

Mooney, G 2010, 'Reasoning, not recipes: helping your students develop statistical understanding and enjoy the experience!' *Australian Mathematics Teacher*, 66(2), pp. 22–24.

Mulligan, J 2015, 'Moving beyond basic numeracy: data modelling in the early years of schooling', *ZDM Mathematics Education*, 47(4), pp. 653–663.

Nicholson, J Ridgway, J & McCusker, S 2006, 'Reasoning with data – time for a rethink?' *Teaching Statistics*, 28(1), pp. 2–9.

Watson, JM 2001, 'Profiling teachers' competence and confidence to teach particular mathematics topics: the case of chance and data', *Journal of Mathematics Teacher Education*, 4(4), pp. 305–337.

Watson, JM 2013, *Statistical Literacy at School: growth and goals*, Routledge.

Watson, J & Callingham, R 2003, 'Statistical literacy: a complex hierarchical construct', *Statistics Education Research Journal*, 2(2), pp. 3–46.

Watson, JM & Callingham, RA 2004, 'Statistical literacy: from idiosyncratic to critical thinking', *Curricular Development in Statistics Education*, International Association for Statistical Education (IASE) Roundtable, Lund, Sweden, pp. 116–162.

Watson, J 2007, 'The foundations of chance and data', *Australian Primary Mathematics Classroom*, vol. 12, no. 1, pp. 4–7.

Watson, JM 2005, 'Variation and expectation as foundations for the chance and data curriculum', in P Clarkson, A Downton, D Gronn, M Horne, A McDonough, R Pierce & A Roche (eds), *Building Connections: theory, research and practice: proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia*, Melbourne, MERGA, Sydney, pp. 35–42.

Representing data

Burrill, G & Elliott, PC 2006, *Thinking and Reasoning with Data and Chance*, National Council of Teachers of Mathematics, Reston, Va.

Carmichael, C, Callingham, R, Watson, J & Hay, I 2009, 'Factors influencing the development of middle school students' interest in statistical literacy', *Statistics Education Research Journal*, 8(1), p. 62.

Callingham, R & Watson, JM 2004, 'Measuring statistical literacy', *Journal of Applied Measurement*, 6(1), pp. 19–47.

Garfield, JB & Ben-Zvi, D 2008, *Developing Students' Statistical Reasoning: connecting research and teaching practice*, Springer, Dordrecht; London.

Konold, C & Kazak, S 2008, 'Reconnecting data and chance', *Technology Innovations in Statistics Education*, 2(1).

MacGillivray, H & Pereira-Mendoza, L 2011, 'Teaching statistical thinking through investigative projects', in *Teaching Statistics in School Mathematics: challenges for teaching and teacher education*, Springer, Netherlands, pp. 109–120.

Mooney, G 2010, 'Reasoning, not recipes: helping your students develop statistical understanding and enjoy the experience!' *Australian Mathematics Teacher*, 66(2), pp. 22–24.

Mulligan, J 2015, 'Moving beyond basic numeracy: data modelling in the early years of schooling', *ZDM Mathematics Education*, 47(4), pp. 653–663.

Nicholson, J, Ridgway, J & McCusker, S 2006, 'Reasoning with data – time for a rethink?' *Teaching Statistics*, 28(1), pp. 2–9.

Watson, JM 2001, 'Profiling teachers' competence and confidence to teach particular mathematics topics: the case of chance and data', *Journal of Mathematics Teacher Education*, 4(4), pp. 305–337.

Watson, JM 2006, *Statistical Literacy at School: growth and goals*, Lawrence Erlbaum Associates, Mahwah, New Jersey.

Watson, J & Callingham, R 2003, 'Statistical literacy: a complex hierarchical construct', *Statistics Education Research Journal*, 2(2), pp. 3–46.

Watson, JM & Callingham, RA 2004, 'Statistical literacy: from idiosyncratic to critical thinking', *Curricular Development in Statistics Education*, International Association for Statistical Education (IASE) Roundtable, Lund, Sweden, pp. 116–162.

Appendix 5. Numeracy experts consulted during development

ACARA acknowledges the contribution of the following numeracy experts who were consulted during the development of the progression:

- Associate Professor Catherine Attard
- Professor Janette Bobis
- Professor Merrilyn Goos
- Dr Peter Gould
- Dr Sally Howell
- Christine Mae
- Professor Joanne Mulligan
- Professor Dianne Siemon
- Emeritus Professor Kaye Stacey
- Associate Professor Jennifer Stephenson
- Emeritus Professor Jane Watson